Quiz 6

February 10, 2017

Show all work and circle your final answer.

1. (7 points) Evalute a $\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$.

$$= \int \frac{5\cos\theta\, d\theta}{(5\sin\theta)^2\sqrt{25\cos^2\theta}}$$

$$= \int \frac{\cancel{5\cos\theta}\, d\theta}{25\sin^2\theta(\cancel{5\cos\theta})}$$

$$= \frac{1}{25}\int \frac{d\theta}{\sin^2\theta}$$

$$= \frac{1}{25}\int \csc^2\theta\, d\theta$$

$$= \frac{-1}{25}\cot\theta + C = \boxed{\frac{-\sqrt{25-x^2}}{25x} + C}$$

$x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$25 - x^2 = 25 - (5\sin\theta)^2$

$= 25 - 25\sin^2\theta$

$= 25(1 - \sin^2\theta$

$= 25\cos^2\theta$

$\sin\theta = \frac{x}{5}$

2. (13 points) Evaluate $\int_0^{2\sqrt{3}} \frac{x^3}{(x^2+4)^{3/2}}\,dx$

$$= \int_{\theta=0}^{\theta=\pi/3} \frac{(2\tan\theta)^3}{(4\sec^2\theta)^{3/2}} \cdot 2\sec^2\theta\, d\theta$$

$$= \int_0^{\pi/3} \frac{8\tan^3\theta}{8\sec^3\theta} \cdot 2\sec^2\theta\, d\theta$$

$$= 2\int_0^{\pi/3} \frac{\tan^3\theta}{\sec\theta}\, d\theta$$

split into sin/cos

$$= 2\int_0^{\pi/3} \frac{\sin^3\theta}{\cos^3\theta}\,\frac{\cos\theta}{1}\, d\theta$$

$$= 2\int_0^{\pi/3} \frac{\sin^3\theta}{\cos^2\theta}\, d\theta$$

$$= 2\int_0^{\pi/3} \frac{(1-\cos^2\theta)\sin\theta\, d\theta}{\cos^2\theta}$$

$u = \cos\theta$

$$= -2\int_1^{1/2} \frac{1-u^2}{u^2}\, du$$

$$= 2\int_{1/2}^{1} \frac{1-u^2}{u^2}\, du$$

$$= 2\int_{1/2}^{1} u^{-2} - 1\, du$$

$$= 2\left[-\frac{1}{u} - u\right]_{1/2}^{1}$$

$$= 2\left(-1 - 1 + 2 + \tfrac{1}{2}\right) = \boxed{1}$$

OR

use $\tan^2\theta = \sec^2\theta - 1$

$$= 2\int_0^{\pi/3} \frac{\tan\theta(\sec^2\theta - 1)}{\sec\theta}\, d\theta$$

$$= 2\int_0^{\pi/3} \sec\theta\tan\theta - \frac{\tan\theta}{\sec\theta}\, d\theta$$

$$= 2\int_0^{\pi/3} \sec\theta\tan\theta - \sin\theta\, d\theta$$

$$= 2\left[\sec\theta + \cos\theta\right]_0^{\pi/3}$$

$$= 2\left(2 + \tfrac{1}{2} - 1 - 1\right)$$

$$= \boxed{1}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$x^2 + 4 = (2\tan\theta)^2 + 4$

$= 4\tan^2\theta + 4$

$= 4(\tan^2\theta + 1)$

$= 4\sec^2\theta$

bounds:

$2\tan\theta = 2\sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = \pi/3$

$2\tan\theta = 0$

$\tan\theta = 0$

$\theta = 0$